Strength envelope for continental crust

Using a spreadsheet or other computer program, calculate strength (failure) envelopes for continental crust using the following parameters. Be sure to show all of your work, and email a copy of your spreadsheet or program to both Matt and Ramses.

- Crustal thickness = 40 km
- Average crustal density = 2850 kg·m$^{-3}$
- Surface temperature = 10 °C = 283 K
- Thermal gradient = 25 K·km$^{-1}$

1) In the brittle (frictional) regime, the strength of most crustal rocks is not dependent upon temperature, strain rate, and mineralogy (with the exception of clays). In this deformation regime, the yield strength of a rock can be expressed as:

$$\Delta \sigma = \sigma_1 - \sigma_3 = \alpha \rho g z (1 - \lambda)$$

Where:
- $\Delta \sigma$ = stress difference (strength)
- $\sigma_1$ = maximum principal compressive stress
- $\sigma_3$ = minimum principal compressive stress
- $\alpha$ = scaling factor for friction coefficient and faulting type
- $\rho$ = density
- $g$ = gravitational constant (9.8 m·s$^{-2}$)
- $z$ = depth
- $\lambda$ = pore pressure/hydrostatic pressure ratio

This expression is a form of Byerlee’s law.

Calculate the frictional strength envelope of the crust ($\Delta \sigma$ as a function of depth) assuming $\alpha=0.8$ and $\lambda=0.4$. Include hand calculations for $\Delta \sigma$ at depths of 10 km and 20 km to confirm your spreadsheet values.
2) In the ductile (plastic) regime, the strength of most crustal rocks is dependent upon temperature, strain rate, and mineralogy. In this deformation regime, rock strength can be expressed as a power-law rheology in the form:

\[
\Delta \sigma = \sigma_1 - \sigma_3 = (\varepsilon/A_D)^{1/n} e^{(E/nRT)}
\]

\(\varepsilon\) = strain rate (s\(^{-1}\))
\(A_D\) = “Dorn parameter”
\(n\) = stress exponent
\(E\) = activation energy
\(R\) = universal gas constant = 8.314 J·K\(^{-1}\)·mol\(^{-1}\)
\(T\) = temperature (K)

Calculate the ductile strength envelope assuming values appropriate for wet granite:
\(\varepsilon = 10^{-12}\) s\(^{-1}\)
\(A_D = 100\) GPa\(^{-n}\)·s\(^{-1}\) (note that \(n\) is not a unit in this case, but rather part of the power law)
\(n = 1.9\)
\(E = 1.37 \times 10^5\) J·mol\(^{-1}\)

Include hand calculations for \(\Delta \sigma\) at depths of 10 km and 20 km to confirm your spreadsheet values.

3) **Plot both strength envelopes on a single graph and show the strength envelope for the crust.** Note that for a given depth, the yield strength is given by the weakest of the two failure mechanisms.

4) **What is the depth and temperature of the brittle-ductile transition in this model?** Compare these values to your intuition – does your answer make sense for Earth?